Nuclear Structure Effects in "Elastic" Neutrino-Induced Reactions*

B. GOULARD AND H. PRIMAKOFF
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
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The effects of nuclear structure on "elastic" neutrino-induced reactions: $\nu_\mu+[Z,A]\rightarrow\mu^-+[Z+1,A]$, $\bar{\nu}_\mu+[Z,A]\rightarrow\mu^++[Z-1,A]$, are considered by use of methods analogous to those developed in the theory of muon capture by complex nuclei. A "nuclear structure effect quantity" ("n.s.e.q."), which allows calculation of the cross sections for $\nu_\mu+n\rightarrow\mu^-+p$ and $\bar{\nu}_\mu+p\rightarrow\mu^++n$ from the observed cross sections for $\nu_\mu+[Z,A]_0\rightarrow\mu^-+[Z+1,A]_{\rm all}$ and $\bar{\nu}_\mu+[Z,A]_0\rightarrow\mu^++[Z-1,A]_{\rm all}$, is derived and discussed. This "n.s.e.q." is, to a reasonable approximation, independent of the nucleon form factors associated with the lepton-hadron weak interaction and essentially involves only the nucleon-nucleon correlation function of $[Z,A]_0$ as determined by the internucleon forces and the exclusion principle. Finally, it is shown how the nuclear parameters entering into the nucleon-nucleon correlation function can be found from existing empirical data on electron scattering and muon capture by complex nuclei and, with use of the so-determined values of these parameters, values of the "n.s.e.q." for various $[Z,A]$ and E_ν are given.

INTRODUCTION

In the present work we wish to calculate the effects of nuclear structure on the "elastic" neutrino-induced reactions[1]

$$\nu_\mu+[Z,A]_0\rightarrow\mu^-+[Z+1,A]_k, \quad (1)$$

$$\bar{\nu}_\mu+[Z,A]_0\rightarrow\mu^++[Z-1,A]_k, \quad (2)$$

where $[Z,A]_0$ represents the spin-unaligned target nucleus in its ground state and $[Z\pm1,A]_k$ the residual nucleus in either its ground state ($k=0$) or in an excited state ($k>0$). The excited state can be either unbound or bound; in the former case $[Z\pm1,A]_k$ decays almost immediately by nucleon emission, e.g., $[Z+1,A]_k\rightarrow[Z,A-1]_0+p$, while in the latter case $[Z\pm1,A]_k$ decays relatively slowly by photon emission, e.g., $[Z+1,A]_k\rightarrow[Z+1,A]_0+\gamma$. The effects of nuclear structure which we calculate are responsible for the difference between

$$d\sigma([Z,A]_0\rightarrow[Z+1,A]_{\rm all};E_\nu)/d(\cos\theta)$$

and $(A-Z)d\sigma(n\sum p;E_\nu)/d(\cos\theta)$, and for the difference between

$$d\sigma([Z,A]_0\rightarrow[Z-1,A]_{\rm all};E_\nu)/d(\cos\theta)$$

and $Zd\sigma(p\rightarrow n;E_\nu)/d(\cos\theta)$, where

$$[Z\pm1,A]_{\rm all}\equiv\sum_{k=0}^{k=k_{\rm max}}[Z\pm1,A]_k,$$

$$[0,1]_k=[0,1]_0\equiv n,\quad [1,1]_k=[1,1]_0\equiv p,$$

and where the various $d\sigma/d(\cos\theta)$ are differential cross sections for the indicated "elastic" neutrino-induced reactions.[2] It is clear from the existence of inhibitions in the transition rate arising from the exclusion principle for nucleons that these differences are: (1) greatest for neutrino-muon three-momentum transfers $|\mathbf{p}_\nu-\mathbf{p}_\mu|\ll$ nucleon Fermi momentum p_F ($\cong265$ MeV/c), (2) vanish when $|\mathbf{p}_\nu-\mathbf{p}_\mu|\gg2p_F$. It is also clear that for any $|\mathbf{p}_\nu-\mathbf{p}_\mu|$, the exact value of the differences depends in a rather complicated way on the nucleon-nucleon correlation function of $[Z,A]_0$ as determined by the internucleon forces and the exclusion principle.

CROSS SECTIONS IN CLOSURE APPROXIMATION

In our analysis of the "elastic" neutrino-induced reactions, we shall suppose that the various possible "inelastic" neutrino-induced reactions, e.g.,[2]

$$\nu_\mu+[Z,A]_0\rightarrow\mu^-+[Z+1,A]_\kappa\rightarrow\mu^-+[Z+1,A]_k+\pi^0, \quad (3)$$

$$\bar{\nu}_\mu+[Z,A]_0\rightarrow\mu^++[Z-1,A]_\kappa\rightarrow\mu^++[Z-1,A]_k+\pi^0, \quad (4)$$

can be distinguished experimentally from the corresponding "elastic" reactions [Eqs. (1), (2)][3] and that the neglect of their effect in the calculation of the cross

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[1] Experimental observations of the reactions in Eqs. (1) and (2) are described by the Columbia-Brookhaven group: G. Danby, J. M. Gaillard, K. Goulianos, L. M. Lederman, M. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962); and by the CERN group: reports at the International Conference on Weak Interactions, Brookhaven, 1963 (unpublished) and at the International Conference on Elementary Particles, Sienna, 1963 (unpublished)—see especially J. S. Bell, J. Løvseth, and M. Veltman, CERN 7509/TH. 382, 1963 (unpublished). The importance of nuclear structure effects in such reactions was, to our knowledge, first emphasized by V. L. Telegdi. A calculation of these effects, based on a Fermi gas model of the nucleus, has been given by S. Berman, CERN 61-22 (unpublished). Our results (see below) are numerically not too different from Berman's. A calculation similar in many ways to Berman's has been published by J. Løvseth, Phys. Letters **5**, 199 (1963).

[2] We exclude from the "elastic" category the neutrino-induced formation of such highly excited states of $[Z\pm1,A]$ that their subsequent decay occurs by pion emission. We label such states $[Z\pm1,A]_\kappa$ and picture them as predominantly, $\{[Z,A-1]+(\mathfrak{N}^*)^+\}_\kappa$, $\{[Z-1,A-1]+(\mathfrak{N}^*)^0\}_\kappa$ where $\mathfrak{N}^*$ is the (100-MeV wide!) $\frac{3}{2}$, $\frac{3}{2}$ nucleon isobar. Thus, the elementary particle processes underlying the "inelastic" neutrino-induced reactions of Eqs. (3) and (4) are taken to be: $\nu_\mu+n\rightarrow\mu^-+(\mathfrak{N}^*)^+$, $(\mathfrak{N}^*)^+\rightarrow p+\pi^0$, $\bar{\nu}_\mu+p\rightarrow\mu^++(\mathfrak{N}^*)^0$, $(\mathfrak{N}^*)^0\rightarrow n+\pi^0$.

[3] We ignore the "elastic" neutrino-induced reactions on atomic electrons of the target: $\nu_\mu+e^-\rightarrow\mu^-+\nu_e$ which yield muons in forward directions ($0\leqq\theta\leqq m_e/m_\mu$) for $E_\nu>(m_\mu^2-m_e^2)/2m_e=10$ BeV.

sections of the "elastic" reactions does not introduce an appreciable error. This last assumption is not unreasonable since the primitive lepton-hadron interaction is weak and since, at neutrino energies of present experimental interest,[1] both "elastic" and "inelastic" cross sections are far below the unitarity limit.

We treat the "elastic" neutrino-induced reactions by means of the "impulse approximation" as regards the superposition of the contributions of the various nucleons and the closure approximation as regards the $\sum_{k=0}^{k=k_{\max}}\cdots$. This procedure is reminiscent of that employed in the theory of capture of orbital negative muons by complex nuclei[4] where the basic reaction

$$\mu^- + [Z,A]_0 \to \nu_\mu + [Z-1,\, A]_k\,, \tag{5}$$

is essentially the lepton-transposed version of the reaction in Eq. (2). Therefore, in accordance with the methods used in the theory of muon capture,[4] we can write the transition operator associated with the "elastic" neutrino-induced reactions as

$$T^{(\pm)} \cong \sum_{i=1}^{A} T_i^{(\pm)} = \sum_{i=1}^{A} \frac{G}{\sqrt{2}} L_\alpha e^{i\mathbf{q}\cdot\mathbf{r}_i} \tau_i^{(\pm)} [B_\alpha^{(\pm)}(\mathbf{q},q_0)]_i\,, \tag{6}$$

where

$$L_\alpha \equiv \gamma_4\gamma_\alpha(1+\gamma_5)\,;\quad G=10^{-5}/m_p^2\,;$$

$$[B_\alpha^{(\pm)}(\mathbf{q},q_0)]_i \equiv \left(\kappa F_V(q^2)[\gamma_4\gamma_\alpha]_i + \tfrac{1}{2}\mu F_M(q^2)\frac{q_\beta}{m_p}[\gamma_4\sigma_{\beta\alpha}]_i - ib'F_S(q^2)\frac{q_\alpha}{m_\mu}[\gamma_4]_i\right)$$

$$\pm\left(\lambda F_A(q^2)[\gamma_4\gamma_\alpha\gamma_5]_i + \tfrac{1}{2}\mu' F_E(q^2)\frac{q_\beta}{m_p}[\gamma_4\sigma_{\beta\alpha}\gamma_5]_i - ibF_P(q^2)\frac{q_\alpha}{m_\mu}[\gamma_4\gamma_5]_i\right);$$

$$q_\alpha \equiv (p_\nu - p_\mu)_\alpha,\quad q^2 \equiv q_\alpha q_\alpha = (\mathbf{q})^2 - (q_0)^2 = (\mathbf{p}_\nu - \mathbf{p}_\mu)^2 - (E_\nu - E_\mu)^2 = -m_\mu^2 + 2E_\nu E_\mu(1 - (|\mathbf{p}_\mu|/E_\mu)\cos\theta)\,; \tag{7}$$

$$\gamma_\alpha = \gamma_\alpha^\dagger,\quad \sigma_{\alpha\beta} \equiv (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2i,\quad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4\,;\quad \langle p|\tau^{(+)}|n\rangle = \langle n|\tau^{(-)}|p\rangle = 1\,.$$

In Eqs. (6), (7), $F_V(q^2)$, $F_M(q^2)$, $F_S(q^2)$, $F_A(q^2)$, $F_E(q^2)$, $F_P(q^2)$ are polar-vector, weak-magnetism, induced-scalar, axial-vector, weak-electricity, and induced-pseudoscalar nucleon weak form factors[5] with normalization: $F_V(0) = F_M(0) = F_S(0) = F_A(0) = F_E(0) = F_P(0) = 1$. According to the conserved polar-vector current hypothesis, applied to a local lepton-hadron weak interaction, $\kappa = 1$, $\mu =$ isovector nucleon anomalous magnetic moment $= 1.79 - (-1.91) = 3.70$, $b' = 0$, while $F_V(q^2)$ and $F_M(q^2)$ are equal, respectively, to the now empirically known Dirac and Pauli isovector nucleon electromagnetic form factors $F_1(q^2)$, $F_2(q^2)$; if, on the other hand, the lepton-hadron weak interaction is mediated by a spin-one boson of mass m_W, we have $F_V(q^2)/F_1(q^2) = F_M(q^2)/F_2(q^2) = (1+q^2/m_W^2)^{-1}$ while κ, μ, b' are still 1, 3.70, 0. As regards the form factors associated with the axial-vector current, nuclear beta-decay data, together with the assumption of $\mu - e$ universality, give $\lambda = 1.21$, the hypothesis of pion-pole dominance of $F_P(q^2)$ yields

$$|b| = \left|\frac{m_\mu}{m_\pi^2}(m_\pi[F_A(-m_\pi^2)]_{\pi\to\mathrm{vac}})(\sqrt{2}g_{\pi pp})\right| = 14 = \left(1 + \frac{0.9m_\mu^2}{m_\pi^2}\right)(8\lambda)$$

and $F_P(q^2) = (1+q^2/m_\pi^2)^{-1}$, while the assumption that the axial-vector current is odd under "isoparity" $(Gj_\alpha^{(A)}G^{-1} = -j_\alpha^{(A)})$ implies $\mu' = 0$. Thus, we consider only the dependence on q^2 of $F_A(q^2)$ as essentially unknown at present; however, none of our results below on the numerical magnitude of the nuclear structure effect quantity is at all sensitive to assumptions made regarding the values of the nucleon weak form factors. Finally, the "impulse approximation" type equality, $T^{(\pm)} \cong \sum_{i=1}^{A} T_i^{(\pm)}$, implies neglect of the contribution of pion-exchange currents to $T^{(\pm)}$; the order of magnitude of this contribution is[6]

$$\pm f^4\left(\frac{G}{\sqrt{2}}L_\alpha \lambda F_A(q^2)\right) \sum_{i=1,\,j=1}^{A} (\tau_j^{(\pm)} - \tau_i^{(\pm)})(e^{i\mathbf{q}\cdot\mathbf{r}_i}[\gamma_4\gamma_\alpha\gamma_5]_i - e^{i\mathbf{q}\cdot\mathbf{r}_j}[\gamma_4\gamma_\alpha\gamma_5]_j)\exp(-m_\pi r_{ij})$$

$$\approx \left\{\frac{4f^4 A}{(m_\pi R)^3}\right\} \sum_{i=1}^{A} T_i^{(\pm)} \ll \sum_{i=1}^{A} T_i^{(\pm)}\,, \tag{8}$$

where $R \equiv$ radius of nucleus $= (0.85/m_\pi)A^{1/3}$ and $f \equiv$ pion-nucleon (pseudovector) coupling constant $= (0.08)^{1/2}$.

[4] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959); R. Klein and L. Wolfenstein, Phys. Rev. Letters **9**, 408 (1962); J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. **41**, 236 (1963); J. S. Bell and J. Løvseth, CERN 7315/TH. 379, 1963 (unpublished).

[5] L. Wolfenstein, Nuovo Cimento **10**, 882 (1958); M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958); S. Weinberg, *ibid.* **112**, 1375 (1958); A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).

[6] See R. J. Blin-Stoyle, V. Gupta, and H. Primakoff, Nucl. Phys. **11**, 444 (1959), and J. S. Bell and R. J. Blin-Stoyle, *ibid.* **6**, 87 (1958) for a discussion of the contribution of pion-exchange currents in beta decay.

Such pion-exchange contributions to $T^{(\pm)}$ are associated with processes of the type $\nu_\mu+n_1\to\mu^-+n_1+\pi^+$; $\pi^++n_2\to p_2$, where the n_1, n_2 are a pair of neutrons of the target nucleus and the π^+ is virtual. The contributions to $T^{(\pm)}$ of analogous processes with a real π^+ have not yet been estimated in detail, but may well be small since real π^+ exchanges are presumably dominated by $\nu_\mu+n_1\to\mu^-+\{(\mathfrak{N}^*)^+\}_1$; $\{(\mathfrak{N}^*)^+\}_1\to n_1+\pi^+$; $\pi^++n_2\to\{(\mathfrak{N}^*)^+\}_2$; $\cdots$, with a real π^+ eventually emerging outside the nucleus.

With the transition operator specified as in Eqs. (6) and (7), we can proceed to write down the differential cross sections $d\sigma([Z,A]_0\to[Z\pm1]_{\text{all}};E_\nu)/d(\cos\theta)$. We have, with ϵ_k, ϵ_0, the energies characterizing the nuclear states $|[Z\pm1,A]_k\rangle\equiv|k\rangle$, $|[Z,A]_0\rangle\equiv|0\rangle$, and with $E_f=\{E_\mu\}_k+\epsilon_k$, the energy of the final state of the $\{\mu^\mp+[Z\pm1,A]_k\}$ system,

$$\frac{d\sigma([Z,A]_0\to[Z\pm1,A]_{\text{all}};E_\nu)}{d(\cos\theta)}=2\pi\sum_{k=0}^{k=k_{\max}}\frac{\{[\{|\mathbf{p}_\mu|\}_k]^2/\{\partial E_f/\partial|\mathbf{p}_\mu|\}_k\}2\pi}{(2\pi)^3}$$
$$\times\sum_{\text{spins}}\left|\frac{G}{\sqrt{2}}\langle\mu|L_\alpha|\nu\rangle\langle k|\sum_{i=1}^{A}\{\Lambda\}_k e^{i\{\mathbf{q}\}_k\cdot\mathbf{r}_i}\tau_i^{(\pm)}[B_\alpha^{(\pm)}(\{\mathbf{q}\}_k,\{q_0\}_k)]_i|0\rangle\right|^2, \quad (9)$$

where, with neglect of terms of order $(m_\mu^2/[\{E_\mu\}_k]^2)\ll1$,

$$\{|\mathbf{p}_\mu|\}_k=([\{E_\mu\}_k]^2-m_\mu^2)^{1/2}\cong\{E_\mu\}_k=[E_\nu-(\epsilon_k-\epsilon_0)];$$

$$\{\mathbf{q}\}_k=\mathbf{p}_\nu-\hat{p}_\mu\{|\mathbf{p}_\mu|\}_k\cong E_\nu 2\sin\frac{\theta}{2}\left[\frac{\hat{p}_\nu-\hat{p}_\mu}{|\hat{p}_\nu-\hat{p}_\mu|}\right]+(\epsilon_k-\epsilon_0)\hat{p}_\mu;\quad \{q_0\}_k=E_\nu-\{E_\mu\}_k=\epsilon_k-\epsilon_0;$$

$$\{q^2\}_k=-m_\mu^2+2E_\nu\{E_\mu\}_k\left[1-\left(\frac{\{|\mathbf{p}_\mu|\}_k}{\{E_\mu\}_k}\right)\cos\theta\right]\cong4E_\nu^2\sin^2\frac{\theta}{2}\left(1-\frac{(\epsilon_k-\epsilon_0)}{E_\nu}\right), \quad (10)$$

and where $\{\Lambda\}_k$ is a projection operator for positive energy nuclear state $|k\rangle$,

$$\{\Lambda\}_k\equiv\frac{H+|\epsilon_k|}{2|\epsilon_k|};\quad H|k\rangle=\epsilon_k|k\rangle;\quad \epsilon_k\leqq(\epsilon_k)_{\max}=E_\nu-m_\mu+\epsilon_0. \quad (11)$$

In the spirit of the above-mentioned "impulse approximation," we assume that the replacement

$$\{\Lambda\}_k\approx[\Lambda(\{\mathbf{q}\}_k,\{q_0\}_k)]_i\equiv\frac{\{[i\gamma_4\boldsymbol{\gamma}]_i\cdot\{\mathbf{q}\}_k+[\gamma_4]_i m_p\}+|\{q_0\}_k+m_p|}{2|\{q_0\}_k+m_p|} \quad (12)$$

does not appreciably distort the transition nuclear matrix element in Eq. (9) for those nuclear states $|k\rangle$ which contribute importantly and, that for these importantly contributing states $|k\rangle$, the quantity $\{\partial E_f/\partial|\mathbf{p}_\mu|\}_k$ can be evaluated as if the nucleon participating in the neutrino $\to$ muon transformation is unbound and originally stationary,[7] viz.,

$$E_f=\{E_\mu\}_k+\{q_0\}_k+\epsilon_0\approx\{E_\mu\}_k+([\{\mathbf{q}\}_k]^2+m_p^2)^{1/2}-m_p+\epsilon_0,$$

$$\left\{\frac{\partial E_f}{\partial|\mathbf{p}_\mu|}\right\}_k\approx\frac{\{|\mathbf{p}_\mu|\}_k}{\{E_\mu\}_k}+\frac{(-|\mathbf{p}_\nu|\cos\theta+\{|\mathbf{p}_\mu|\}_k)}{([\{\mathbf{q}\}_k]^2+m_p^2)^{1/2}}=\left(1+\frac{2E_\nu}{m_p}\sin^2\frac{\theta}{2}\right)\left[\frac{m_p}{\{q_0\}_k+m_p}\right]. \quad (13)$$

Then, extending the $\sum_{k=0}^{k=k_{\max}}\cdots$ over *all* states $|k\rangle$ in accord with the closure approximation, we obtain from Eqs. (9), (10), (12), and (13),

$$\frac{d\sigma([Z,A]_0\to[Z\pm1,A]_{\text{all}};E_\nu)}{d(\cos\theta)}\cong\frac{(\langle E_\mu\rangle)^2[(\langle q_0\rangle+m_p)/m_p]}{(2\pi)[1+(2E_\nu/m_p)(\sin\frac{1}{2}\theta)^2]}\frac{G^2}{2}L_{\beta\alpha}B_{\beta\alpha}^{(\pm)}, \quad (14)$$

[7] The approximation of treating the nucleon participating in the neutrino $\to$ muon transformation as originally stationary is not serious since the momenta of the various nucleons $\{\mathbf{p}_{\mathfrak{N}}\}_0$ are uniformly distributed about $\mathbf{p}_\nu$, so that terms linear in $\{\mathbf{p}_{\mathfrak{N}}\}_0\cdot\{\mathbf{q}\}_k$ pretty well cancel out on the average. With particular reference to Eqs. (17)–(19), (42), where the above approximation is used to evaluate $\langle\epsilon_k-\epsilon_0\rangle/E_\nu$, we note in addition that this last quantity makes a relatively small contribution to $\langle E_\mu\rangle$, $\langle q^2\rangle$, $|\langle\mathbf{q}\rangle|$.

with

$$L_{\beta\alpha}\equiv\{\sum_{\text{spins}}\langle\nu|L_\beta|\mu\rangle\langle\mu|L_\alpha|\nu\rangle\}$$
$$=2(\langle E_\mu\rangle E_\nu)^{-1}[(\langle p_\mu\rangle)_\beta(p_\nu)_\alpha+(\langle p_\mu\rangle)_\alpha(p_\nu)_\beta-\delta_{\beta\alpha}(\langle p_\mu\rangle)_\lambda(p_\nu)_\lambda+\epsilon_{\beta\alpha\eta\lambda}(\langle p_\mu\rangle)_\eta(p_\nu)_\lambda](1-2\delta_{\beta 4}),\quad(15)$$

$$B_{\beta\alpha}{}^{(\pm)}\equiv\langle 0|\sum_{j=1,i=1}^{A}e^{-i\langle\mathbf{q}\rangle\cdot\mathbf{r}_{ji}}\tau_j{}^{(\mp)}\tau_i{}^{(\pm)}[B_\beta{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_j{}^\dagger[\Lambda(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_j[\Lambda(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i[B_\alpha{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i|0\rangle$$
$$=\langle 0|\sum_{i=1}^{A}\tfrac{1}{2}(1\mp\tau_i{}^{(3)})[B_\beta{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i{}^\dagger[\Lambda(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i[B_\alpha{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i|0\rangle$$
$$+\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})e^{-i\langle\mathbf{q}\rangle\cdot\mathbf{r}_{ji}}\tfrac{1}{4}[(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\pm i(\boldsymbol{\tau}_j\times\boldsymbol{\tau}_i)^{(3)}]$$
$$\times[B_\beta{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_j{}^\dagger[\Lambda(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_j[\Lambda(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i[B_\alpha{}^{(\pm)}(\langle\mathbf{q}\rangle,\langle q_0\rangle)]_i|0\rangle\equiv[B_{\beta\alpha}{}^{(\pm)}]_{\text{I}}+[B_{\beta\alpha}{}^{(\pm)}]_{\text{II}},\quad(16)$$

where the average indicated by $\langle\cdots\rangle$

$$\langle E_\mu\rangle=E_\nu\left(1-\frac{\langle\epsilon_k-\epsilon_0\rangle}{E_\nu}\right);\quad\langle\mathbf{q}\rangle\cong E_\nu\left\{2\sin\frac{\theta}{2}\left[\frac{(\hat{p}_\nu-\hat{p}_\mu)}{|\hat{p}_\nu-\hat{p}_\mu|}\right]+\frac{\langle\epsilon_k-\epsilon_0\rangle}{E_\nu}\hat{p}_\mu\right\},$$
$$\langle q_0\rangle=\langle\epsilon_k-\epsilon_0\rangle;\quad\langle q^2\rangle\cong 4E_\nu{}^2(\sin\tfrac{1}{2}\theta)^2(1-\langle\epsilon_k-\epsilon_0\rangle/E_\nu),\quad(17)$$

is taken over the various states $|k\rangle$. Again in the spirit of the "impulse approximation," we shall evaluate $\langle E_\mu\rangle$, $\langle\mathbf{q}\rangle$, $\langle q_0\rangle$, $\langle q^2\rangle$ on the assumption that, in the predominant states $|k\rangle$, the nucleon participating in the neutrino $\rightarrow$ muon transformation moves as if it were unbound and originally stationary.[7] We then have

$$\frac{\langle\epsilon_k-\epsilon_0\rangle}{E_\nu}\cong\frac{2(E_\nu/m_p)[\sin\frac{1}{2}\theta]^2}{1+(2E_\nu/m_p)[\sin\frac{1}{2}\theta]^2},\quad(18)$$

which upon substitution in Eq. (17) immediately yields $\langle E_\mu\rangle$, $\langle\mathbf{q}\rangle$, $\langle q_0\rangle$, $\langle q^2\rangle$; in particular

$$\langle E_\mu\rangle\cong E_\nu\Big/\left[1+\frac{2E_\nu}{m_p}(\sin\theta)^2\right];\quad\langle q^2\rangle\cong\frac{4E_\nu{}^2(\sin\frac{1}{2}\theta)^2}{1+(2E_\nu/m_p)(\sin\frac{1}{2}\theta)^2}.\quad(19)$$

We proceed to the evaluation of $L_{\beta\alpha}[B_{\beta\alpha}{}^{(\pm)}]_{\text{I}}$ and of $L_{\beta\alpha}[B_{\beta\alpha}{}^{(\pm)}]_{\text{II}}$ as defined in Eqs. (16), (15), (12), and (7). As will be seen, it is in the evaluation of $[B_{\beta\alpha}{}^{(\pm)}]_{\text{II}}$ that considerations of nuclear physics enter into the calculation of $d\sigma([Z,A]_0\rightarrow[Z\pm1,A]_{\text{all}};E_\nu)/d(\cos\theta)$. First of all, using a nonrelativistic, i.e., 2^A component approximation for the state $|0\rangle$, we have,

$$L_{\beta\alpha}[B_{\beta\alpha}{}^{(\pm)}]_{\text{I}}=4(\cos\tfrac{1}{2}\theta)^2[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\left[\frac{m_p}{\langle q_0\rangle+m_p}\right]\Phi_{\text{I}}{}^{(\pm)}(\langle q^2\rangle,\theta);$$

$$\Phi_{\text{I}}{}^{(\pm)}(\langle q^2\rangle,\theta)\equiv[F_V(\langle q^2\rangle)]^2+\lambda^2[F_A(\langle q^2\rangle)]^2+(\tan\tfrac{1}{2}\theta)^2$$
$$\times\left\{2\lambda^2[F_A(\langle q^2\rangle)]^2\pm\frac{E_\nu}{m_p}4\lambda[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]F_A(\langle q^2\rangle)-\frac{m_\mu}{m_p}\lambda bF_A(\langle q^2\rangle)F_P(\langle q^2\rangle)\right\}$$
$$+(\langle q^2\rangle/2m_p{}^2)(\tan\tfrac{1}{2}\theta)^2\{[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]^2+\tfrac{1}{2}(\cot\tfrac{1}{2}\theta)^2[\mu F_M(\langle q^2\rangle)]^2$$
$$+\lambda^2[F_A(\langle q^2\rangle)]^2\mp2\lambda[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]F_A(\langle q^2\rangle)+\tfrac{1}{2}b^2[F_P(\langle q^2\rangle)]^2\}.\quad(20)$$

It is to be noted that for $Z=0$, $A=1$: $\nu_\mu+n\rightarrow\mu^-+p$, or for $Z=1$, $A=1$: $\bar{\nu}_\mu+p\rightarrow\mu^++n$, the quantity $[B_{\beta\alpha}{}^{(\pm)}]_{\text{II}}$ vanishes and, since the initial n or p is here at rest, the nonrelativistic approximation for $|0\rangle$ is precise; thus, in this case, Eqs. (14)–(20) give an exact description of $d\sigma(n\to p;E_\nu)/d(\cos\theta)$ and $d\sigma(p\to n;E_\nu)/d(\cos\theta)$, viz.,

$$\begin{aligned}\frac{d\sigma(n\to p;E_\nu)}{d(\cos\theta)}&=\left(\frac{G^2m_p^2}{\pi}\right)\frac{(E_\nu/m_p)^2(\cos\frac{1}{2}\theta)^2}{(1+2(E_\nu/m_p)(\sin\frac{1}{2}\theta)^2)^3}\Phi_{\mathrm{I}}^{(+)}(\langle q^2\rangle,\theta)\,,\\ \frac{d\sigma(p\to n;E_\nu)}{d(\cos\theta)}&=\left(\frac{G^2m_p^2}{\pi}\right)\frac{(E_\nu/m_p)^2(\cos\frac{1}{2}\theta)^2}{(1+2(E_\nu/m_p)(\sin\frac{1}{2}\theta)^2)^3}\Phi_{\mathrm{I}}^{(-)}(\langle q^2\rangle,\theta)\,,\end{aligned}\tag{21}$$

in agreement with formulas already published.[8]

We now describe the evaluation of $L_{\beta\alpha}[B_{\beta\alpha}^{(\pm)}]_{\mathrm{II}}$ using again a nonrelativistic approximation for $|0\rangle$. A lengthy but straightforward calculation based on Eqs. (16), (15), (12), and (7) yields, to a sufficient approximation,

$$L_{\beta\alpha}[B_{\beta\alpha}^{(\pm)}]_{\mathrm{II}}\cong-4(\cos\tfrac{1}{2}\theta)^2[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\left[\frac{m_p}{\langle q_0\rangle+m_p}\right]\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)\,;$$

$$\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)\equiv-[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]^{-1}\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})e^{-i\langle\mathbf{q}\rangle\cdot\mathbf{r}_{ji}}\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j^{(3)}\tau_i^{(3)})$$
$$\times\left\{G_1(\langle q^2\rangle,\theta)+G_2^{(\pm)}(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i}{3}+G_3(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p^2}\right\}|0\rangle\,;$$

$$G_1(\langle q^2\rangle,\theta)\equiv[F_V(\langle q^2\rangle)]^2-(\langle q^2\rangle/2m_p^2)F_V(\langle q^2\rangle)[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]\,,$$

$$\begin{aligned}G_2^{(\pm)}(\langle q^2\rangle,\theta)\equiv&\left\{1+2(\tan\tfrac{1}{2}\theta)^2+\frac{\langle q^2\rangle}{2m_p^2}(1+3(\tan\tfrac{1}{2}\theta)^2)-(1+(\tan\tfrac{1}{2}\theta)^2)\frac{E_\nu}{2m_p}\left(1-\frac{\langle E_\mu\rangle}{E_\nu}\cos\theta\right)\left(1+\frac{E_\nu}{\langle E_\mu\rangle}\right)\right\}\lambda^2[F_A(\langle q^2\rangle)]^2\\ &\pm\left(\frac{E_\nu}{m_p}-\frac{\langle q^2\rangle}{4m_p^2}\right)(\tan\tfrac{1}{2}\theta)^2 4\lambda[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]F_A(\langle q^2\rangle)-\frac{m_\mu}{\langle E_\mu\rangle}(1+(\tan\tfrac{1}{2}\theta)^2)\frac{E_\nu}{2m_p}\left(1-\frac{\langle E_\mu\rangle}{E_\nu}\cos\theta\right)\\ &\times\lambda bF_A(\langle q^2\rangle)F_P(\langle q^2\rangle)+(\langle q^2\rangle/4m_p^2)(1+3(\tan\tfrac{1}{2}\theta)^2)[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]^2\,,\end{aligned}\tag{22}$$

$$\begin{aligned}G_3(\langle q^2\rangle,\theta)\equiv&[F_V(\langle q^2\rangle)]^2-(\tan\tfrac{1}{2}\theta)^2[F_V(\langle q^2\rangle)+\mu F_M(\langle q^2\rangle)]^2\\ &+[1-(\tan\tfrac{1}{2}\theta)^2-(1+(\tan\tfrac{1}{2}\theta)^2)(2m_p/\langle E_\mu\rangle)]\lambda^2[F_A(\langle q^2\rangle)]^2\\ &+(m_\mu/\langle E_\mu\rangle)(1+(\tan\tfrac{1}{2}\theta)^2)\lambda bF_A(\langle q^2\rangle)F_P(\langle q^2\rangle)+(\tan\tfrac{1}{2}\theta)^2b^2[F_P(\langle q^2\rangle)]^2\,,\end{aligned}$$

where the cylindrical symmetry about the direction of $\mathbf{p}_\nu$ of the processes $\nu_\mu+[Z,A]_0\to\mu^-+[Z+1,A]_{\mathrm{all}}$, $\bar\nu_\mu+[Z,A]_0\to\mu^++[Z-1,A]_{\mathrm{all}}$ has been employed to simplify somewhat the expression for $\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)$. Equations (14)–(22) give:

$$\begin{aligned}\frac{d\sigma([Z,A]_0\to[Z\pm1,A]_{\mathrm{all}};E_\nu)}{d(\cos\theta)}&=[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\left\{\frac{G^2m_p^2}{\pi}\frac{(E_\nu/m_p)^2(\cos\frac{1}{2}\theta)^2}{[1+2(E_\nu/m_p)(\sin\frac{1}{2}\theta)^2]^3}\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\right\}\left\{1-\frac{\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)}{\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)}\right\}\\ &=[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\{d\sigma(\substack{n\to p\\ p\to n};E_\nu)/d(\cos\theta)\}\left\{1-\frac{\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)}{\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)}\right\},\end{aligned}\tag{23}$$

so that the nuclear structure effect is completely described by the quantity $\{1-\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\}$ with $\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)$ and $\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)$ defined in Eqs. (22), (20). Division of any observed $d\sigma([Z,A]_0\to[Z\pm1,A]_{\mathrm{all}};E_\nu)/d(\cos\theta)$ by a theoretically calculated $[A(\frac{1}{2}\pm\frac{1}{2})\mp Z]\{1-\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\}$ will yield the desired $d\sigma(\substack{n\to p\\ p\to n};E_\nu)/d(\cos\theta)$.

The remainder of our discussion is devoted to a calculation of $\{1-\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\}$; we shall see that, to a reasonable degree of approximation, $\{1-\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\}$ is independent of the nucleon weak form factors $F_V(\langle q^2\rangle)$, $F_M(\langle q^2\rangle)$, $F_A(\langle q^2\rangle)$, and $F_P(\langle q^2\rangle)$ and essentially involves only the nucleon-nucleon correlation functions of $[Z,A]_0$ as determined by the internucleon forces and the exclusion principle.

[8] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960); Phys. Rev. 119, 1410 (1960); 126, 2239 (1962); Y. Yamaguchi, CERN 61-2 (unpublished); Progr. Theoret. Phys. (Kyoto) 23, 1117 (1960); N. Cabbibo and R. Gatto, Nuovo Cimento 15, 304 (1960).

EVALUATION OF NUCLEAR STRUCTURE EFFECT QUANTITY

To evaluate the nuclear structure effect quantity $\{1-\Phi_{II}{}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_I{}^{(\pm)}(\langle q^2\rangle,\theta)\}$, we express $\Phi_{II}{}^{(\pm)}(\langle q^2\rangle,\theta)$ as

$$\begin{aligned}\Phi_{II}{}^{(\pm)}(\langle q^2\rangle,\theta)=&-[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]^{-1}\Bigg\{\Bigg[\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\tfrac{1}{2}(1+P_{ji})\\&\times\left\{G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i}{3}+G_3(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}\right\}|0\rangle\Bigg]h_+{}^{(\pm)}(\langle\mathbf{q}\rangle)\\&+\Bigg[\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\tfrac{1}{2}(1-P_{ji})\\&\times\left\{G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i}{3}+G_3(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}\right\}|0\rangle\Bigg]h_-{}^{(\pm)}(\langle\mathbf{q}\rangle)\Bigg\},\end{aligned}\tag{24}$$

$$P_{ji}\equiv-(\tfrac{1}{2}(1+\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i))(\tfrac{1}{2}(1+\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i)),$$

$$h_\pm{}^{(\pm)}(\langle\mathbf{q}\rangle)\equiv\int\int e^{-i\langle\mathbf{q}\rangle\cdot(\mathbf{r}'-\mathbf{r})}\mathcal{H}_\pm{}^{(\pm)}(\mathbf{r}',\mathbf{r})d\mathbf{r}'d\mathbf{r},$$

$$\mathcal{H}_\pm{}^{(\pm)}(\mathbf{r}',\mathbf{r})\equiv\frac{\begin{aligned}\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\Big\{G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i}{3}\\+G_3(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}\Big\}\delta(\mathbf{r}'-\mathbf{r}_j)\delta(\mathbf{r}-\mathbf{r}_i)\left(\frac{1\pm P_{ji}}{2}\right)|0\rangle\end{aligned}}{\begin{aligned}\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\Big\{G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i}{3}\\+G_3(\langle q^2\rangle,\theta)\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}\Big\}\left(\frac{1\pm P_{ji}}{2}\right)|0\rangle\end{aligned}},$$

where the $\mathcal{H}_\pm{}^{(\pm)}(\mathbf{r}',\mathbf{r})$ are the indicated nucleon-nucleon correlation functions of $[Z,A]_0$. We then use[9]

$$\begin{aligned}&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})|0\rangle=-\tfrac{1}{2}A+\langle 0|\mathbf{T}^2-[T^{(3)}]^2|0\rangle\cong-\tfrac{1}{2}A+\tfrac{1}{2}|A-2Z|,\\&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\tfrac{1}{3}(\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i)|0\rangle=-\tfrac{1}{2}A+\tfrac{1}{3}\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle\cong-\tfrac{1}{2}A+\tfrac{1}{2}|A-2Z|,\\&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}|0\rangle\cong\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}[-\tfrac{1}{2}A+\tfrac{1}{2}|A-2Z|],\\&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})P_{ji}|0\rangle=-\tfrac{1}{2}[Z(A-Z)]+\langle 0|\mathbf{S}_p{}^2+\mathbf{S}_n{}^2-(\mathbf{S}_p+\mathbf{S}_n)^2|0\rangle,\\&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\tfrac{1}{3}(\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i)P_{ji}|0\rangle=-\tfrac{1}{2}[Z(A-Z)]-\tfrac{1}{3}\langle 0|\mathbf{S}_p{}^2+\mathbf{S}_n{}^2-(\mathbf{S}_p+\mathbf{S}_n)^2|0\rangle,\\&\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j{}^{(3)}\tau_i{}^{(3)})\frac{\boldsymbol{\sigma}_j\cdot\langle\mathbf{q}\rangle\boldsymbol{\sigma}_i\cdot\langle\mathbf{q}\rangle}{4m_p{}^2}P_{ji}|0\rangle\cong\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}[-\tfrac{1}{2}(Z(A-Z))-\tfrac{1}{3}\langle 0|\mathbf{S}_p{}^2+\mathbf{S}_n{}^2-(\mathbf{S}_p+\mathbf{S}_n)^2|0\rangle],\end{aligned}\tag{25}$$

[9] See Eqs. (7b)–(7e) in Primakoff, Ref. 4. The approximate equality $\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle\cong\tfrac{3}{2}|A-2Z|$ is a consequence of supermultiplet theory; see Appendix. This theory, first developed before the war [see E. P. Wigner, Phys. Rev. **51**, 106 (1937); E. P. Wigner and E. Feenberg, Rept. Progr. Phys. **8**, 274 (1941)] has received considerable recent support. Thus, J. D. Anderson and C. Wong, Phys. Rev. Letters **7**, 250 (1961), and A. M. Lane and J. M. Soper, Nucl. Phys. **37**, 663 (1962), show that the isospin quantum number is quite good in the ground state of a variety of medium-heavy nuclei while P. Franzini and L. A. Radicati, Phys. Letters **6**, 322 (1963), carry out calculations of ground-state binding energy of a large number of nuclei in the framework of the supermultiplet approximation and find excellent agreement with experiment.

so that, substituting Eq. (25) into Eq. (24),

$$\Phi_{\rm II}{}^{(\pm)}(\langle q^2\rangle,\theta)=\Bigg\{\bigg[G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)+G_3(\langle q^2\rangle,\theta)\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}\bigg]\frac{A}{2[A(\frac{1}{2}\pm\frac{1}{2})\mp Z]}$$
$$-\bigg[G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)+G_3(\langle q^2\rangle,\theta)\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}\bigg]\frac{|A-2Z|}{2[A(\frac{1}{2}\pm\frac{1}{2})\mp Z]}\Bigg\}\left(\frac{h_+{}^{(\pm)}(\langle\mathbf{q}\rangle)+h_-{}^{(\pm)}(\langle\mathbf{q}\rangle)}{2}\right)$$
$$+\Bigg\{\bigg[G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)+G_3(\langle q^2\rangle,\theta)\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}\bigg]\frac{Z(A-Z)}{2[A(\frac{1}{2}\pm\frac{1}{2})\mp Z]}$$
$$-\bigg[G_1(\langle q^2\rangle,\theta)-\tfrac{1}{3}G_2{}^{(\pm)}(\langle q^2\rangle,\theta)-G_3(\langle q^2\rangle,\theta)\frac{|\langle\mathbf{q}\rangle|^2}{4m_p{}^2}\bigg]\frac{\frac{1}{3}\langle 0|\mathbf{S}_p{}^2+\mathbf{S}_n{}^2-(\mathbf{S}_p+\mathbf{S}_n)^2|0\rangle}{[A(\frac{1}{2}\pm\frac{1}{2})\mp Z]}\Bigg\}$$
$$\times\left(\frac{h_+{}^{(\pm)}(\langle\mathbf{q}\rangle)-h_-{}^{(\pm)}(\langle\mathbf{q}\rangle)}{2}\right).\qquad(26)$$

On the other hand, Eqs. (22) and (20) show that

$$\Phi_{\rm I}{}^{(\pm)}(\langle q^2\rangle,\theta)=[G_1(\langle q^2\rangle,\theta)+G_2{}^{(\pm)}(\langle q^2\rangle,\theta)+G_3(\langle q^2\rangle,\theta)(\langle q^2\rangle/4m_p{}^2)],\qquad(27)$$

whence, with neglect of terms of relative order $(\langle q^2\rangle/4m_p{}^2)^2$, $(\langle q^2\rangle/4m_p{}^2)(|A-2Z|/A)$, and $\langle 0|\mathbf{S}_p{}^2+\mathbf{S}_n{}^2-(\mathbf{S}_p+\mathbf{S}_n)^2|0\rangle/Z(A-Z)$, the nuclear structure effect quantity $\{1-\Phi_{\rm II}{}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\rm I}{}^{(\pm)}(\langle q^2\rangle,\theta)\}$ assumes the rather simple form

$$\{1-\Phi_{\rm II}{}^{(+)}(\langle q^2\rangle,\theta)/\Phi_{\rm I}{}^{(+)}(\langle q^2\rangle,\theta)\}=\left\{1-\left(\frac{A}{2(A-Z)}-\frac{|A-2Z|}{2(A-Z)}\right)\eta_+{}^{(+)}(\langle\mathbf{q}\rangle)-(\tfrac{1}{2}Z)\eta_-{}^{(+)}(\langle\mathbf{q}\rangle)\right\},$$

$$\{1-\Phi_{\rm II}{}^{(-)}(\langle q^2\rangle,\theta)/\Phi_{\rm I}{}^{(-)}(\langle q^2\rangle,\theta)\}=\left\{1-\left(\frac{A}{2Z}-\frac{|A-2Z|}{2Z}\right)\eta_+{}^{(-)}(\langle\mathbf{q}\rangle)-(\tfrac{1}{2}(A-Z))\eta_-{}^{(-)}(\langle\mathbf{q}\rangle)\right\},\qquad(28)$$

$$\eta_\pm{}^{(\pm)}(\langle\mathbf{q}\rangle)\equiv\tfrac{1}{2}(h_+{}^{(\pm)}(\langle\mathbf{q}\rangle)\pm h_-{}^{(\pm)}(\langle\mathbf{q}\rangle))\equiv\int\int e^{-i\langle\mathbf{q}\rangle\cdot(\mathbf{r}'-\mathbf{r})}\tfrac{1}{2}[\mathcal{H}_+{}^{(\pm)}(\mathbf{r}',\mathbf{r})\pm\mathcal{H}_-{}^{(\pm)}(\mathbf{r}',\mathbf{r})]d\mathbf{r}'d\mathbf{r},$$

with the nucleon-nucleon correlation functions of $[Z,A]_0$, the $\mathcal{H}_\pm{}^{(\pm)}(\mathbf{r}',\mathbf{r})$, given in Eqs. (24), (22). Equations (23) and (28) yield

$$\frac{d\sigma([Z,A]_0\to[Z+1,A]_{\rm all};E_\nu)}{d(\cos\theta)}=(A-Z)\frac{d\sigma(n\to p;E_\nu)}{d(\cos\theta)}\left\{1-\left(\frac{A}{2(A-Z)}-\frac{|A-2Z|}{2(A-Z)}\right)\eta_+{}^{(+)}(\langle\mathbf{q}\rangle)-(\tfrac{1}{2}Z)\eta_-{}^{(+)}(\langle\mathbf{q}\rangle)\right\},$$

$$\frac{d\sigma([Z,A]_0\to[Z-1,A]_{\rm all};E_\nu)}{d(\cos\theta)}=Z\frac{d\sigma(p\to n;E_\nu)}{d(\cos\theta)}\left\{1-\left(\frac{A}{2Z}-\frac{|A-2Z|}{2Z}\right)\eta_+{}^{(-)}(\langle\mathbf{q}\rangle)-(\tfrac{1}{2}(A-Z))\eta_-{}^{(-)}(\langle\mathbf{q}\rangle)\right\},\qquad(29)$$

which exhibits the general character of the nuclear structure effect.

NUMERICAL PREDICTION OF NUCLEAR STRUCTURE EFFECT QUANTITY

Various remarks can be made with regard to the over-all form of Eqs. (28) and (29) before one embarks on a calculation of the $\eta_\pm{}^{(\pm)}(\langle\mathbf{q}\rangle)$. Thus, it is reasonable to take

$$\mathcal{H}_\pm{}^{(+)}(\mathbf{r}',\mathbf{r})\cong\mathcal{H}_\pm{}^{(-)}(\mathbf{r}',\mathbf{r})\equiv\mathcal{H}_\pm(\mathbf{r}',\mathbf{r}),\qquad(30)$$

since $G_2{}^{(\pm)}(\langle q^2\rangle,\theta)$ appears in both numerator and denominator of the defining Eq. (24), and since the $\pm$ term in $G_2{}^{(\pm)}(\langle q^2\rangle,\theta)$ [Eq. (22)] is relatively important only at large θ (i.e., at $|\langle\mathbf{q}\rangle|\approx E_\nu$) where $\eta_\pm{}^{(\pm)}(\langle\mathbf{q}\rangle)$ is itself quite small. Equations (30) and (28) give

$$\eta_\pm{}^{(+)}(\langle\mathbf{q}\rangle)\cong\eta_\pm{}^{(-)}(\langle\mathbf{q}\rangle)\equiv\eta_\pm(\langle\mathbf{q}\rangle)\qquad(31)$$

so that Eq. (29) yields for nuclei with $A-Z=Z$,

$$\left\{\frac{d\sigma([Z,A]_0\to[Z+1,A]_{\rm all};E_\nu)}{d(\cos\theta)}\Bigg/\frac{d\sigma([Z,A]_0\to[Z-1,A]_{\rm all};E_\nu)}{d(\cos\theta)}\right\}\cong\left\{\frac{d\sigma(n\to p;E_\nu)}{d(\cos\theta)}\Bigg/\frac{d\sigma(p\to n;E_\nu)}{d(\cos\theta)}\right\},\qquad(32)$$

the nuclear structure effect canceling out in the ratio of cross sections. Further, in the limit of $\langle\mathbf{q}\rangle\to 0$, we have from Eqs. (28) and (24): $\eta_+^{(\pm)}=1$, $\eta_-^{(\pm)}=0$, so that Eq. (29) becomes (for $A\geqq 2Z$)

$$\frac{d\sigma([Z,A]_0\to[Z+1,A]_{\mathrm{all}};E_\nu)}{d(\cos\theta)}=(A-Z)\frac{d\sigma(n\to p;E_\nu)}{d(\cos\theta)}\{1-Z/(A-Z)\}=(A-2Z)\frac{d\sigma(n\to p;E_\nu)}{d(\cos\theta)}$$
$$\frac{d\sigma([Z,A]_0\to[Z-1,A]_{\mathrm{all}};E_\nu)}{d(\cos\theta)}=Z\frac{d\sigma(p\to n;E_\nu)}{d(\cos\theta)}\{1-Z/Z\}=0. \tag{33}$$

Equation (33) demonstrates in a rather striking way the effect of the Pauli exclusion principle on the "elastic" neutrino-induced reaction cross sections in the limit of vanishing three-momentum transfer. We also mention that Eq. (28) shows that $\eta_\pm^{(\pm)}(\langle\mathbf{q}\rangle)$ becomes very small for $|\langle\mathbf{q}\rangle|\gg\{|\mathbf{r}'-\mathbf{r}|_{\mathrm{av}}\}^{-1}$ so that the nuclear structure effect quantity $\{1-\Phi_{\mathrm{II}}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}^{(\pm)}(\langle q^2\rangle,\theta)\}$ approaches 1 under these circumstances.

We now proceed to an explicit calculation of the $\eta_\pm(\langle\mathbf{q}\rangle)$ from Eqs. (28), (30), and (31). For this purpose we assume that $\mathfrak{K}_\pm(\mathbf{r}',\mathbf{r})$ of Eqs. (30), (24), and (22) can be written as[10]

$$\mathfrak{K}_\pm(\mathbf{r}',\mathbf{r})\cong\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j^{(3)}\tau_i^{(3)})\delta(\mathbf{r}'-\mathbf{r}_j)\delta(\mathbf{r}-\mathbf{r}_i)(\tfrac{1}{2}(1\pm P_{ji}))|0\rangle\Big/$$
$$\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\tfrac{1}{4}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i-\tau_j^{(3)}\tau_i^{(3)})(\tfrac{1}{2}(1\pm P_{ji}))|0\rangle \tag{34}$$
$$\cong\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})\delta(\mathbf{r}'-\mathbf{r}_j)\delta(\mathbf{r}-\mathbf{r}_i)(\tfrac{1}{2}(1\pm P_{ji}))|0\rangle\Big/\langle 0|\sum_{j=1,i=1}^{A}(1-\delta_{ji})(\tfrac{1}{2}(1\pm P_{ji}))|0\rangle$$

and then further approximated by[11]

$$\mathfrak{K}_\pm(\mathbf{r}',\mathbf{r})\approx(1\pm\delta_0/A)^{-1}\mathfrak{D}(|\mathbf{r}'|)\mathfrak{D}(|\mathbf{r}|)(1\pm f(|\mathbf{r}'-\mathbf{r}|)),$$
$$\delta_0/A\equiv\int\int\mathfrak{D}(|\mathbf{r}'|)\mathfrak{D}(|\mathbf{r}|)f(|\mathbf{r}'-\mathbf{r}|)d\mathbf{r}'d\mathbf{r}, \tag{35}$$
$$\mathfrak{D}(|\mathbf{r}|)\equiv A^{-1}\langle 0|\sum_{i=1}^{A}\delta(\mathbf{r}-\mathbf{r}_i)|0\rangle.$$

Equations (28), (31), and (35) give

$$\eta_+(\langle\mathbf{q}\rangle)=(1-\delta_0^2/A^2)^{-1}\left[\Delta(\langle\mathbf{q}\rangle)-\frac{\delta_0}{A}\frac{\delta(\langle\mathbf{q}\rangle)}{A}\right];$$
$$\eta_-(\langle\mathbf{q}\rangle)=(1-\delta_0^2/A^2)^{-1}\left[\frac{\delta(\langle\mathbf{q}\rangle)}{A}-\frac{\delta_0}{A}\Delta(\langle\mathbf{q}\rangle)\right];$$
$$\Delta(\langle\mathbf{q}\rangle)\equiv\left|\int\int\int e^{-i\langle\mathbf{q}\rangle\cdot\mathbf{r}}\mathfrak{D}(|\mathbf{r}|)d\mathbf{r}\right|^2; \tag{36}$$
$$\frac{\delta(\langle\mathbf{q}\rangle)}{A}\equiv\int\int e^{-i\langle\mathbf{q}\rangle\cdot(\mathbf{r}'-\mathbf{r})}\mathfrak{D}(|\mathbf{r}'|)\mathfrak{D}(|\mathbf{r}|)f(|\mathbf{r}'-\mathbf{r}|)d\mathbf{r}'d\mathbf{r},$$
$$\delta(0)=\delta_0,$$

[10] Strictly speaking, the nucleon-nucleon correlation functions $\mathfrak{K}_\pm^{(\pm)}(\mathbf{r}',\mathbf{r})$, as defined in Eqs. (24) and (22), do depend on the $G_i(\langle q^2\rangle,\theta)$ $(i=1, 2, 3)$, i.e., do depend on the form factors $F_V(\langle q^2\rangle)$, $F_M(\langle q^2\rangle)$, $F_A(\langle q^2\rangle)$, $F_P(\langle q^2\rangle)$ and on θ, and hence vary with E_ν and with θ. However, the $G_i(\langle q^2\rangle,\theta)$ appear in both numerator and denominator of Eq. (24) so that their influence tends to cancel out. This cancellation is complete in the special case when $|0\rangle=\Phi(\mathbf{r}_1,\mathbf{r}_2,\cdots)\mathrm{X}(\sigma_1^{(3)},\sigma_2^{(3)},\cdots;\tau_1^{(3)},\tau_2^{(3)},\cdots)$ and is still very appreciable in the general case when $|0\rangle=\sum_m a_m\Phi_m(\mathbf{r}_1,\mathbf{r}_2,\cdots)\mathrm{X}_m(\sigma_1^{(3)},\sigma_2^{(3)},\cdots;\tau_1^{(3)},\tau_2^{(3)},\cdots)$.

[11] See Eq. (10) in Primakoff, Ref. 4.

or, with neglect of terms $\approx 1/A^2$,

$$\eta_+(\langle\mathbf{q}\rangle)\cong\Delta(\langle\mathbf{q}\rangle)\,,\qquad \eta_-(\langle\mathbf{q}\rangle)\cong\frac{\delta_0}{A}\left(\frac{\delta(\langle\mathbf{q}\rangle)}{\delta_0}-\Delta(\langle\mathbf{q}\rangle)\right). \tag{37}$$

Substitution of Eqs. (37) and (31) into Eq. (28) then yields

$$\begin{aligned}\{1-\Phi_{\text{II}}^{(+)}(\langle q^2\rangle,\theta)/\Phi_{\text{I}}^{(+)}(\langle q^2\rangle,\theta)\}&=\left\{1-\left(\frac{A}{2(A-Z)}-\frac{|A-2Z|}{2(A-Z)}\right)\Delta(\langle\mathbf{q}\rangle)-\left(\frac{Z}{2A}\right)\delta_0\left(\frac{\delta(\langle\mathbf{q}\rangle)}{\delta_0}-\Delta(\langle\mathbf{q}\rangle)\right)\right\}\\&=\left[1-\left(\frac{Z}{A-Z}\right)\Delta(\langle\mathbf{q}\rangle)\right]\left[1-\frac{(Z/2A)\delta_0(\delta(\langle\mathbf{q}\rangle)/\delta_0-\Delta(\langle\mathbf{q}\rangle))}{1-[Z/(A-Z)]\Delta(\langle\mathbf{q}\rangle)}\right],\end{aligned} \tag{38}$$

$$\begin{aligned}\{1-\Phi_{\text{II}}^{(-)}(\langle q^2\rangle,\theta)/\Phi_{\text{II}}^{(-)}(\langle q^2\rangle,\theta)\}&=\left\{1-\left(\frac{A}{2Z}-\frac{|A-2Z|}{2Z}\right)\Delta(\langle\mathbf{q}\rangle)-\left(\frac{A-Z}{2A}\right)\delta_0\left(\frac{\delta(\langle\mathbf{q}\rangle)}{\delta_0}-\Delta(\langle\mathbf{q}\rangle)\right)\right\}\\&=[1-\Delta(\langle\mathbf{q}\rangle)]\left[1-\frac{[(A-Z)/2A]\delta_0(\delta(\langle\mathbf{q}\rangle)/\delta_0-\Delta(\langle\mathbf{q}\rangle))}{1-\Delta(\langle\mathbf{q}\rangle)}\right],\end{aligned} \tag{39}$$

with the second equality on the right-hand side of Eqs. (38) and (39) valid for $A\geqq 2Z$. It remains to calculate the $\Delta(\langle\mathbf{q}\rangle)$, $\delta(\langle\mathbf{q}\rangle)$ and δ_0 of Eq. (36); to do this we assume[11]

$$\begin{aligned}\mathfrak{D}(|\mathbf{r}|)&=[(4\pi/3)r_0^3A]^{-1}: &&|\mathbf{r}|\leqq r_0A^{1/3}\\&=0 &&:|\mathbf{r}|>r_0A^{1/3}\\f(|\mathbf{r}'-\mathbf{r}|)&=1 &&:|\mathbf{r}'-\mathbf{r}|\leqq d\\&=0 &&:|\mathbf{r}'-\mathbf{r}|>d,\end{aligned} \tag{40}$$

whence

$$\begin{aligned}&\Delta(\langle\mathbf{q}\rangle)=[(3/x)j_1(x)]^2;\quad x\equiv|\langle\mathbf{q}\rangle|r_0A^{1/3};\\&\delta(\langle\mathbf{q}\rangle)\cong\left(\frac{d}{r_0}\right)^3\left\{\frac{3}{y}j_1(y)+\left(\frac{d}{r_0}\right)\left(\frac{9}{4A^{1/3}}\right)\left[\frac{2}{y^4}-\frac{2}{y^2}n_1(y)+\frac{1}{y}n_0(y)\right]\right\};\quad y\equiv|\langle\mathbf{q}\rangle|d;\\&\delta_0\cong\left(\frac{d}{r_0}\right)^3\left(1-\frac{9}{16}\frac{d}{r_0A^{1/3}}\right),\end{aligned} \tag{41}$$

with [see Eqs. (17)–(19)],[7]

$$|\langle\mathbf{q}\rangle|=[\langle q^2\rangle]^{1/2}\left[1+\frac{\langle q^2\rangle}{4m_p^2}\right]^{1/2}=\frac{2E_\nu\sin\frac{1}{2}\theta}{[1+(2E_\nu/m_p)(\sin\frac{1}{2}\theta)^2]^{1/2}}\left[1+\frac{[2E_\nu\sin\frac{1}{2}\theta]^2}{4m_p^2(1+(2E_\nu/m_p)(\sin\frac{1}{2}\theta)^2)}\right]^{1/2}. \tag{42}$$

Comparable results are obtained with other reasonable shapes for $\mathfrak{D}(|\mathbf{r}|)$ and $f(|\mathbf{r}'-\mathbf{r}|)$. It is to be noted that our choice for $\mathfrak{D}(|\mathbf{r}|)$ and $f(|\mathbf{r}'-\mathbf{r}|)$ implies a description of $\mathcal{H}_\pm(\mathbf{r}',\mathbf{r})$ [Eqs. (35) and (34)] and hence of $\eta_\pm(\langle\mathbf{q}\rangle)$, $\Delta(\langle\mathbf{q}\rangle)$, $\delta(\langle\mathbf{q}\rangle)$, δ_0 [Eq. (36)] in terms of a pair of nuclear parameters, r_0 and d, *each of which* is to be determined separately from appropriate experimental data. On the other hand, any calculation of $\mathfrak{D}(|\mathbf{r}|)$ and $f(|\mathbf{r}'-\mathbf{r}|)$ on the basis of an independent-particle nuclear model establishes a definite (and hence restrictive) relationship between parameters like r_0 and d so that *only one of them* is determined separately from experiment; with $Z=A-Z$ this relationship is such that $\delta_0=4$.[12] Thus, in the Fermi gas model with $Z=A-Z$: $r_0=(9\pi/8)^{1/3}(1/p_F)$; $d=4^{1/3}r_0=(9\pi/2)^{1/3}(1/p_F)$;

$$\Delta(\langle\mathbf{q}\rangle)=[(3/x)j_1(x)]^2;$$

$$\delta(\langle\mathbf{q}\rangle)=4\left\{1-\frac{3}{4}\left(\frac{2}{9\pi}\right)^{1/3}y+\frac{1}{16}\left(\frac{2}{9\pi}\right)y^3\right\},\quad\text{for }y\leqq\left(\frac{9\pi}{2}\right)^{1/3}2;\quad\delta(\langle\mathbf{q}\rangle)=0,\quad\text{for }y>\left(\frac{9\pi}{2}\right)^{1/3}2;\quad\delta_0=4,$$

which values for $\Delta(\langle\mathbf{q}\rangle)$, $\delta(\langle\mathbf{q}\rangle)$, δ_0 are to be compared with those in Eq. (41).

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[12] See discussion in Sec. V of C. A. Engelbrecht, Phys. Rev. **133**, B988 (1964).

TABLE I. Values of d/r_0 deduced from muon capture data.

	Z	A	$A-2Z$	$\Delta^{(\mu)}$	δ_0	d/r_0
Ti	22	46–50	2–6	0.47	2.59	1.50
Mo	42	92, 94–98, 100	8, 10–14, 16	0.36	2.85	1.52
Gd	64	152, 154–158, 160	24, 26–30, 32	0.25	3.02	1.53
Pb	82	204, 206–208	40, 42–44	0.21	3.08	1.53

We now discuss the actual numerical values of r_0 and d. Analysis of experimental data on elastic scattering from various nuclei[13] indicates that 1.35×10^{-13} cm$>r_0>1.15\times10^{-13}$ cm for $A>25$; with the value of r_0 so determined, $\Delta(|\langle\mathbf{q}\rangle|)$ can be calculated for any A, E_ν, θ [Eqs. (41) and (42)]. On the other hand, the theory of muon capture in complex nuclei[14] shows that a quantity closely related to the nuclear structure effect quantity

$$\{1-\Phi_{\text{II}}^{(-)}(\langle q^2\rangle,\theta)/\Phi_{\text{I}}^{(-)}(\langle q^2\rangle,\theta)\}$$

of Eq. (39) enters as a multiplicative factor into the theoretical expression for the total capture rate of muons by $[Z,A]_0$—this follows since the transition matrix element associated with $\mu^-+[Z,A]_0\rightarrow\nu_\mu+[Z-1, A]_k$ is essentially the same, apart from a lepton-transposition, as the transition matrix element of Eq. (9) associated with $\bar{\nu}_\mu+[Z,A]_0\rightarrow\mu^++[Z-1, A]_k$. Thus, we can determine the quantity d/r_0 by comparison of theoretical expressions for the muon total capture rates[14] with the corresponding experimental values[15]; such a comparison indicates that:

$$\left[\left\{[1-\Delta^{(\mu)}]\left[1-\frac{[(A-Z)/2A]\delta_0(\delta^{(\mu)}/\delta_0-\Delta^{(\mu)})}{1-\Delta^{(\mu)}}\right]\right\}\cong\left\{[1-\Delta^{(\mu)}]\left[1-\left(\frac{A-Z}{2A}\right)\delta_0\right]\right\}\right]_{\text{theor}}$$
$$=[\{1-[(A-Z)/2A]3.11\}]_{\text{exper}}, \quad (43)$$

where

$$\frac{\delta^{(\mu)}}{A}\equiv\int\int e^{i\langle\mathbf{p}_\nu\rangle\cdot(\mathbf{r}'-\mathbf{r})}\varphi^*(|\mathbf{r}'|)\varphi(|\mathbf{r}|)\mathfrak{D}(|\mathbf{r}'|)\mathfrak{D}(|\mathbf{r}|)f(|\mathbf{r}'-\mathbf{r}|)d\mathbf{r}'d\mathbf{r}\Big/\int|\varphi(|\mathbf{r}|)|^2\mathfrak{D}(|\mathbf{r}|)d\mathbf{r}\cong\frac{\delta_0}{A}\text{ (to within 5\%)},$$
$$\Delta^{(\mu)}\equiv\left|\int e^{i\langle\mathbf{p}_\nu\rangle\cdot\mathbf{r}}\varphi(|\mathbf{r}|)\mathfrak{D}(|\mathbf{r}|)d\mathbf{r}\right|^2\Big/\int|\varphi(|\mathbf{r}|)|^2\mathfrak{D}(|\mathbf{r}|)d\mathbf{r};\quad|\langle\mathbf{p}_\nu\rangle|\cong0.75m_\mu, \quad (44)$$
$$\varphi(|\mathbf{r}|)\equiv\text{orbital muon wave function},$$

with the empirically determined coefficient, 3.11,[15] constant to within a few percent for $A>25$. Equations (43) and (44) fix the values of δ_0 and so of d/r_0 [Eq. (41)] for various Z, A as given in Table I[16]; with d/r_0 and r_0

TABLE II. Calculated values of δ_0.

	Z	A	$A-2Z$	d/r_0	δ_0
				1.47	2.30
Al	13	27	1	1.50	2.43
				1.53	2.56
				1.47	2.49
Fe	26	54, 56–58	2, 4–6	1.50	2.63
				1.53	2.77
				1.47	2.52
Cu	29	63, 65	5, 7	1.50	2.66
				1.53	2.81
				1.47	2.73
Pb	82	204, 206–208	40, 42–44	1.50	2.89
				1.53	3.08

[13] R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables* (Stanford University Press, Stanford, California, 1960).
[14] See Klein and Wolfenstein, Ref. 4. Their numerical values of $\Delta^{(\mu)}$ are used in Table I. See also footnote 70a in R. Winston, Phys. Rev. **129**, 2766 (1963).
[15] J. C. Sens, R. A. Swanson, V. L. Telegdi, and D. D. Yovanovitch, Phys. Rev. **107**, 1464 (1957); J. C. Sens, *ibid.* **113**, 679 (1959); V. L. Telegdi, Phys. Rev. Letters **8**, 327 (1962); T. A. Filippas, P. Palit, R. T. Siegel, and R. E. Welsh, Phys. Letters **6**, 118 (1963).
[16] d/r_0 is also obtainable from an analysis of inelastic (and elastic) electron-nucleus scattering experiments. See B. Goulard, Ph.D. thesis, University of Pennsylvania, 1964 (unpublished).

specified, δ_0 and $\delta(\langle\mathbf{q}\rangle)$ can be calculated for any A, E_ν, θ [Eqs. (41) and (42)]. Values of δ_0 obtained in this way are given in Table II.

We proceed to exhibit a set of curves for the nuclear structure effect quantity ("n.s.e.q.")

$$\{1-\Phi_{II}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{I}^{(\pm)}(\langle q^2\rangle,\theta)\}$$

for various $[Z,A]$ and E_ν, calculated on the basis of Eqs. (38), (39), (41), and (42) and Table II (Figs. 1, 2, and 3). These curves show that $\{1-\Phi_{II}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{I}^{(\pm)}(\langle q^2\rangle,\theta)\}$ (a) increases slowly with increasing r_0; (b) decreases slowly with increasing d; (c) increases rapidly with increasing $|\langle\mathbf{q}\rangle|$, i.e., with increasing θ for fixed E_ν or with increasing E_ν for fixed θ. All of these variations are reasonable from a qualitative point of view; in particular, the curves show that

$$\{1-\Phi_{II}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{I}^{(\pm)}(\langle q^2\rangle,\theta)\}\cong 1,$$

for

$$|\langle\mathbf{q}\rangle|=|\mathbf{p}_\nu-\mathbf{p}_\mu|>\frac{5}{d}\cong\frac{5}{1.5}\frac{1}{r_0}.$$

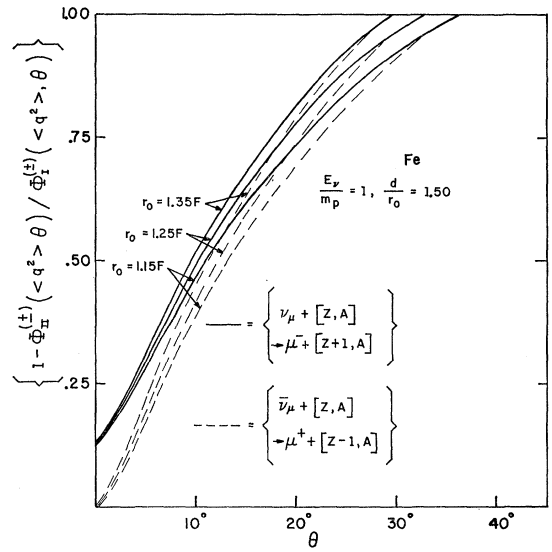

FIG. 1. "N.s.e.q" as ordinate plotted versus $\cos^{-1}(\hat{p}_\mu\cdot\hat{p}_\nu)$ as abscissa, for three values of r_0, with an Fe target, $E_\nu/m_p=1$, and $d/r_0=1.50$. The curves originally contained slight wiggles which have been smoothed out in this figure. The last remark holds also for the following figures.

This last condition is equivalent to $|\langle\mathbf{q}\rangle|>(5/1.5)[(8/9\pi)]^{1/3}p_F\cong 2p_F$, an inequality anticipated in the Introduction on the basis of a physical argument. In general, the curves in Figs. 1, 2, and 3 should be as accurate as the corresponding expressions [Eqs. (43) and (44)] in the theory of muon capture.[4,14,15]

For illustrative purposes we also present a set of curves (Figs. 4 and 5) for

$$\left\langle\frac{d\sigma([Z,A]_0\to[Z\pm1,A]_{\rm all};E_\nu)}{d(\cos\theta)}\right\rangle\equiv\int\frac{d\sigma([Z,A]_0\to[Z\pm1,\,A]_{\rm all};E_\nu)}{d(\cos\theta)}N(E_\nu)dE_\nu$$

$$=[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\int[d\sigma({}^{n\to p}_{p\to n};E_\nu)/d(\cos\theta)]\left\{1-\frac{\Phi_{II}^{(\pm)}(\langle q^2\rangle,\theta)}{\Phi_{I}^{(\pm)}(\langle q^2\rangle,\theta)}\right\}N(E_\nu)dE_\nu \qquad (45)$$

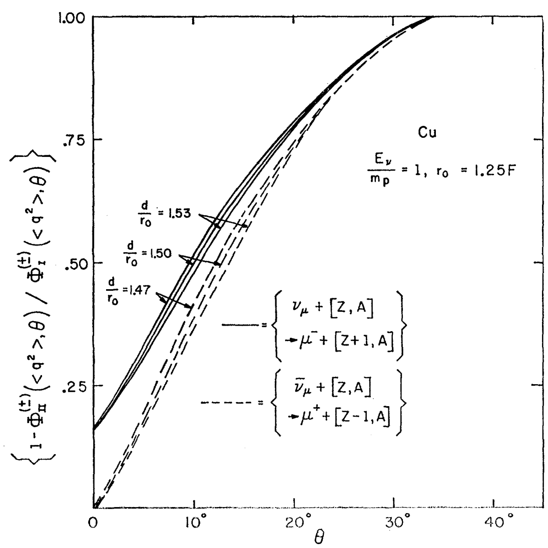

FIG. 2. "N.s.e.q" as ordinate plotted versus $\cos^{-1}(\hat{p}_\mu\cdot\hat{p}_\nu)$ as abscissa, for three values of d/r_0, with a Cu target, $E_\nu/m_p=1$, and $r_0=1.25\times10^{-13}$ cm.

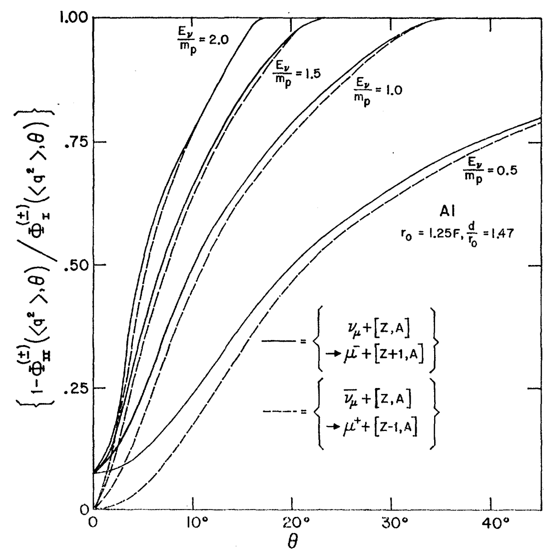

FIG. 3. "N.s.e.q" as ordinate plotted versus $\cos^{-1}(\hat{p}_\mu\cdot\hat{p}_\nu)$ as abscissa, for four values of E_ν, with an Al target, $r_0=1.25\times10^{-13}$ cm and $d/r_0=1.47$.

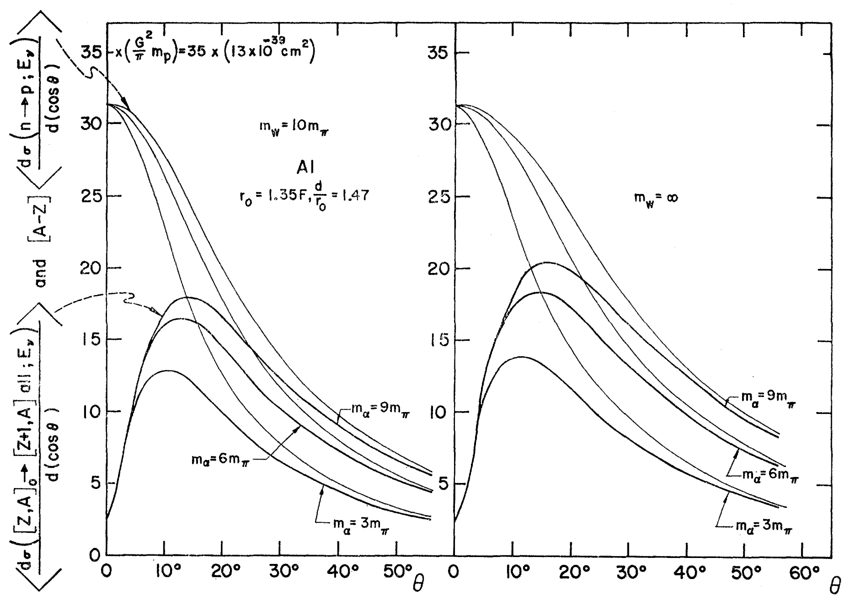

FIG. 4. Differential cross sections of $\{\nu_\mu+n\rightarrow\mu^-+p$ (times $A-Z$) and of $\nu_\mu+[Z,A]_0\rightarrow\mu^-+[Z+1,A]_{\rm all}\}$ averaged over incident neutrino spectrum as ordinate, plotted versus $\cos^{-1}(\hat{p}_\mu\cdot\hat{p}_\nu)$ as abscissa, for various values of m_α, m_W, with Al target, $r_0=1.35\times10^{-13}$ cm, and $d/r_0=1.47$. To obtain differential cross sections per steradian and per neutron divide the vertical scale by $2\pi\times(A-Z)=2\pi\times14$.

[see Eq. (23)] and for

$$[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\langle d\sigma(\genfrac{}{}{0pt}{}{n\rightarrow p}{p\rightarrow n};E_\nu)/d(\cos\theta)\rangle\equiv[A(\tfrac{1}{2}\pm\tfrac{1}{2})\mp Z]\int[d\sigma(\genfrac{}{}{0pt}{}{n\rightarrow p}{p\rightarrow n};E_\nu)/d(\cos\theta)]N(E_\nu)dE_\nu\,, \tag{46}$$

where $N(E_\nu)dE_\nu$ is the Brookhaven incident neutrino energy spectrum[1] and Al is the target element. Our calculated values of the $\{1-\Phi_{\rm II}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\rm I}^{(\pm)}(\langle q^2\rangle,\theta)\}$ are used (Figs. 1, 2, and 3), while the $d\sigma(\genfrac{}{}{0pt}{}{n\rightarrow p}{p\rightarrow n};E_\nu)/d(\cos\theta)$ [Eqs. (21), (20), and (19)] are calculated with $F_V(\langle q^2\rangle)$, $F_M(\langle q^2\rangle)$ and $F_P(\langle q^2\rangle)$ as determined above (i.e., as determined on the basis of the conserved polar-vector current hypothesis and the pion-pole dominance hypothesis[17]) and with

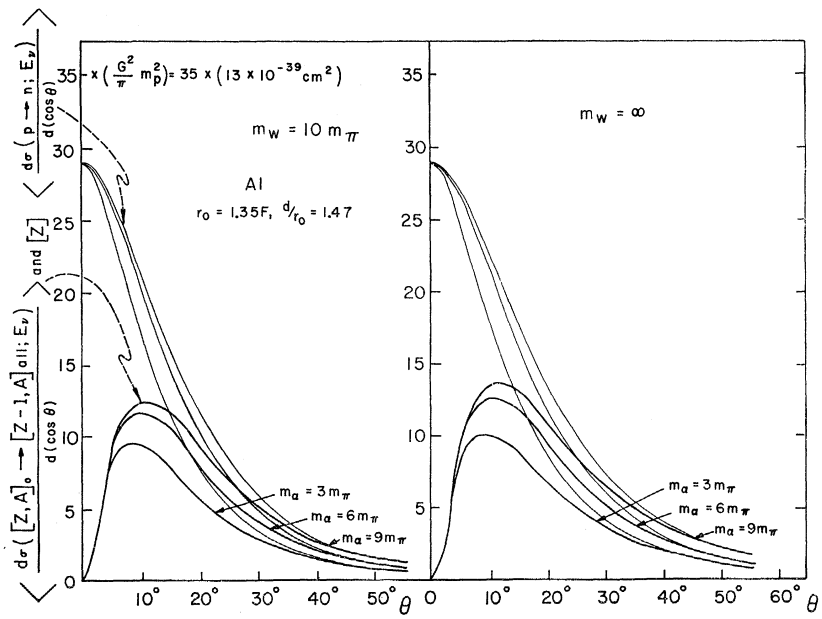

FIG. 5. Differential cross sections of $\{\bar{\nu}_\mu+p\rightarrow\mu^++n$ (times Z) and of $\bar{\nu}_\mu+[Z,A]_0\rightarrow\mu^++[Z-1,A]_{\rm all}\}$ averaged over incident antineutrino spectrum as ordinate, plotted versus $\cos^{-1}(\hat{p}_\mu\cdot\hat{p}_\nu)$ as abscissa for various values of m_α, m_W with an Al target, $r_0=1.35\times10^{-13}$ cm, and $d/r_0=1.47$. (The incident antineutrino spectrum is assumed to be the same as the incident neutrino spectrum.) To obtain differential cross sections per steradian and per proton divide the vertical scale by $2\pi\times Z=2\pi\times13$.

[17] For values of E_ν and θ of interest here, the terms in Eq. (20) [and so in Eq. (21)] containing $[F_P(\langle q^2\rangle)]^2$ and $[F_P(\langle q^2\rangle)][F_A(\langle q^2\rangle)]$ are very small compared to the terms containing $[F_V(\langle q^2\rangle)]^2$, $[F_A(\langle q^2\rangle)]^2$, $[F_M(\langle q^2\rangle)]^2$, $[F_V(\langle q^2\rangle)][F_A(\langle q^2\rangle)]$, $[F_M(\langle q^2\rangle)][F_A(\langle q^2\rangle)]$, and $[F_V(\langle q^2\rangle)][F_M(\langle q^2\rangle)]$.

$F_A(\langle q^2\rangle)$ represented as[18]

$$F_A(\langle q^2\rangle)=(1+\langle q^2\rangle/m_\alpha{}^2)^{-1}(1+\langle q^2\rangle/m_W{}^2)^{-1}; \quad m_\alpha=3m_\pi,\, 6m_\pi,\, 9m_\pi; \quad m_W=10m_\pi,\, \infty\,. \tag{47}$$

It is seen from these curves that, with a reasonable knowledge of the incident neutrino spectrum, differential cross-section measurements of moderate precision of the "elastic" neutrino-induced reactions should be sufficient for an approximate determination of the parameters in $F_A(\langle q^2\rangle)$.

In conclusion, we wish to emphasize that when reliable expressions are available for the nucleon-nucleon correlation function $\mathcal{K}_\pm(\mathbf{r}',\mathbf{r})$ of Eqs. (34), (30), (24), and (22) on the basis, for example, of an analysis of inelastic (and elastic) electron-nucleus scattering experiments, the procedure of Eqs. (35)–(44) and, in particular, the assumptions of Eqs. (35) and (40) regarding $\mathfrak{D}(|\mathbf{r}|)$ and $f(|\mathbf{r}'-\mathbf{r}|)$ will be superfluous. In these circumstances, Eqs. (28), (30), and (31), together with the then empirically known $\mathcal{K}_\pm(\mathbf{r}',\mathbf{r})$, will be immediately applicable to the calculation of $\eta_\pm(\langle\mathbf{q}\rangle)$ and so of the nuclear structure effect quantity $\{1-\Phi_{\mathrm{II}}{}^{(\pm)}(\langle q^2\rangle,\theta)/\Phi_{\mathrm{I}}{}^{(\pm)}(\langle q^2\rangle,\theta)\}$ and the only major remaining approximation in our treatment will refer to the general validity of closure over the residual nuclear states.

ACKNOWLEDGMENT

We are very grateful to Gisèle Goulard for preparing a computer program to obtain the curves in all our figures.

APPENDIX

The state of a nucleus $[Z,A]$ which is a member of a supermultiplet is characterized by isospin and spin quantum numbers $T^{(3)}=\frac{1}{2}[Z-(A-Z)]$, $S^{(z)}=S_p{}^{(z)}+S_n{}^{(z)}$, $Y^{(3),(z)}=S_p{}^{(z)}-S_n{}^{(z)}$ and by three additional quantum numbers P, P', P'' where P is the highest value of any of the three quantities $T^{(3)}$, $S^{(z)}$, $Y^{(3),(z)}$ occurring in the supermultiplet P' is the highest value of a second of these quantities occurring in the supermultiplet which is compatible with the value P of the first, and P'' is the highest value of the third of these quantities occurring in the supermultiplet compatible with the values P, P', of the other two.[9] The ground state of $[Z,A]$ belongs to the supermultiplet characterized by $P=T=|T^{(3)}|=\frac{1}{2}|A-2Z|$, $P'=S_p+S_n$, $P''=S_p-S_n$ where $S_p=0$ or $\frac{1}{2}$ depending on whether Z is even or odd and $S_n=0$ or $\frac{1}{2}$ depending on whether $A-Z$ is even or odd; the only exceptional case corresponds to $Z=A-Z$ with Z odd (e.g., $N_7{}^{14}$, $B_5{}^{10}$, $Li_3{}^6$, $H_1{}^2$) where the above P and P' values must be interchanged (so that $P=1$, $P'=0$, $P''=0$).

To find $\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle$ we consider the expectation value in the state $|0\rangle$ of the "total" space exchange operator $\Xi\equiv\frac{1}{2}\sum_{j=1,i=1}{}^A(1-\delta_{ji})P_{ji}$,

$$\begin{aligned}\langle 0|\Xi|0\rangle&=\langle 0|\tfrac{1}{2}\sum_{j=1,i=1}^{A}(1-\delta_{ji})[-(\tfrac{1}{2}(1+\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i))(\tfrac{1}{2}(1+\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i))]|0\rangle\\ &=-\tfrac{1}{8}A^2+2A-\tfrac{1}{2}\langle 0|(\mathbf{T})^2+(\mathbf{S})^2+(\mathbf{Y}^{[1]})^2+(\mathbf{Y}^{[2]})^2+(\mathbf{Y}^{[3]})^2|0\rangle\\ &=-\tfrac{1}{8}A^2+2A-\tfrac{1}{2}[T(T+1)+2S_p(S_p+1)+2S_n(S_n+1)]-\tfrac{1}{2}\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle,\end{aligned} \tag{A1}$$

while by a group-theoretic argument,[9]

$$\langle 0|\Xi|0\rangle=-\tfrac{1}{8}A^2+2A-\tfrac{1}{2}[P(P+4)+P'(P'+2)+(P'')^2], \tag{A2}$$

whence, with use of the above values of P, P', P'',

$$\langle 0|\Xi|0\rangle=-\tfrac{1}{8}A^2+2A-\tfrac{1}{2}[T(T+4)+2S_p{}^2+2S_n{}^2+2(S_p+S_n)]. \tag{A3}$$

Comparison of Eq. (A1) with Eq. (A3) yields

$$\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle=3T=\tfrac{3}{2}|A-2Z|\,. \tag{A4}$$

This holds for all nuclei except those with $Z=A-Z$ and Z odd; in this case, from Eqs. (A1) and (A2), since $T=0$, $S_p=\frac{1}{2}$, $S_n=\frac{1}{2}$, $P=1$, $P'=0$, $P''=0$,

$$\langle 0|[\mathbf{Y}^{[1]}]^2+[\mathbf{Y}^{[2]}]^2|0\rangle=2. \tag{A5}$$

[18] It is worth mentioning that a speculative field-theoretic argument [see, for example, P. Dennery and H. Primakoff, Phys. Rev. Letters 8, 350 (1962); 8, 466 (1962)] indicates that $\lambda F_A(\langle q^2\rangle)/\kappa F_V(\langle q^2\rangle)=(1.21/1)[F_A(\langle q^2\rangle)/F_V(\langle q^2\rangle)]\to 1$ as $m_\alpha{}^2/\langle q^2\rangle$, $m_\rho{}^2/\langle q^2\rangle$, $\cdots\to 0$.